1) $B = \{(x,y) \in \mathbb{R}^2 : \; x,y>0 \text{ e } x^2+y^2<a\}$

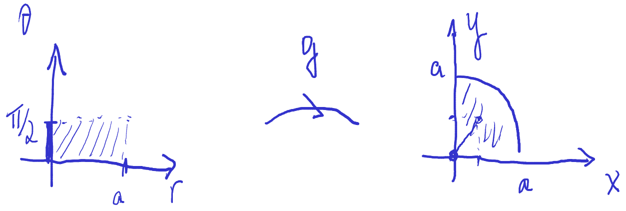
$$\int_B x^2y^3\,dx\,dy = \int_{g(A)} \underbrace{x^2y^3}_{f(x,y)=x^2y^3}\,dx\,dy = \int_A r^2\cos^2\theta \cdot r^3\sin^3\theta \cdot r\,dr\,d\theta = \int_0^{\pi/2}\int_0^a r^6\cos^2\theta\sin^2\theta \cdot \sin\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2}\left(\cos^2\theta\sin^2\theta\sin\theta \cdot \int_0^a r^6\,dr\right)d\theta$$

$$= \frac{a^7}{7}\int_0^{\pi/2}\cos^2\theta(1-\cos^2\theta)\sin\theta\,d\theta \qquad u=\cos\theta \quad du=-\sin\theta\,d\theta$$

$$= \frac{a^7}{7}\int_1^0 -u^2(1-u^2)\,du = \frac{a^7}{7}\int_0^1 u^2-u^4\,du$$

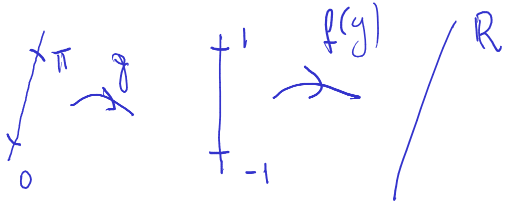
$$= \frac{a^7}{7}\left(\frac{1}{3}-\frac{1}{5}\right).$$



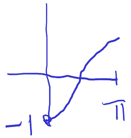
$g(r,\theta) = (r\cos\theta, r\sin\theta)$

$g' = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix} \Rightarrow \det g' = r$

2) $\int_{-1}^{1} \underbrace{\frac{1}{\sqrt{1-y^2}}}_{=f(y)}\,dy$ é imprópria



$y = g(x) = -\cos x, \quad 0 \leq x \leq \pi$



$g(x) = -\cos x$

$g'(x) = \operatorname{sen} x$

$$\int_{-1}^{1} f(y)\,dy = \int_0^{\pi} \frac{1}{\cancel{|\operatorname{sen} x|}} \cdot \cancel{|\operatorname{sen} x|}\,dx = \pi$$

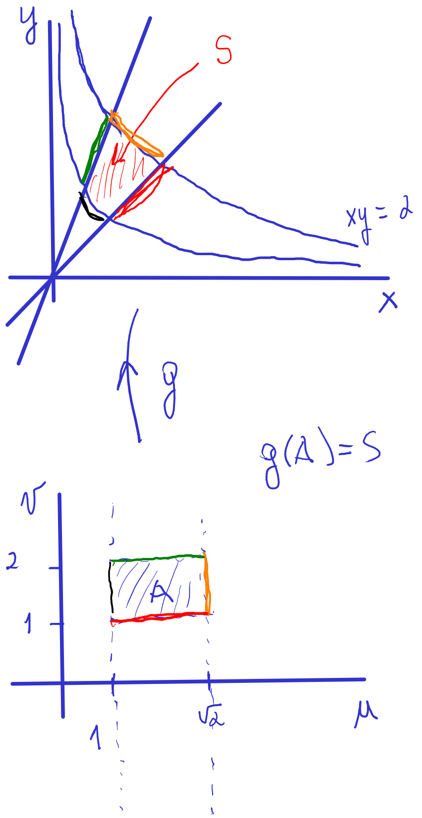
$$\lim_{a\to 1^-}\int_{-a}^{a}\frac{1}{\sqrt{1-y^2}}\,dy$$

20 - lista 2) $S = \{(x,y) \in \mathbb{R}^2 : 1 \leq xy \leq 2,\ x \leq y \leq 4x\}$
$x > 0$

$x = u/v \ ,\ y = uv \Rightarrow g(u,v) = (u/v,\ u \cdot v)$

$g' = \begin{bmatrix} 1/v & -u/v^2 \\ v & u \end{bmatrix} \Rightarrow \det g' = \frac{2u}{v}$

$v = \sqrt{y/x}$ e $u = \sqrt{xy}$



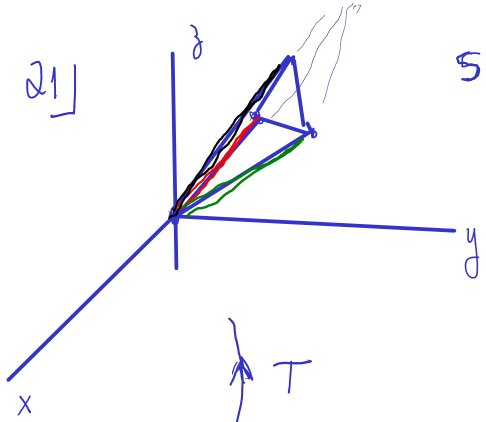
$\int_S x^2 y^3 \,dx\,dy = ?$

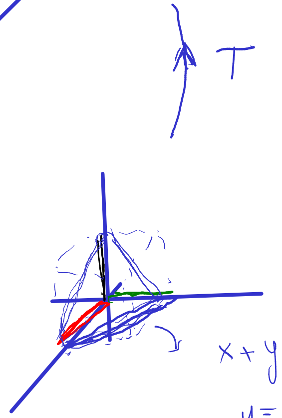
$\| $

$\int_A \frac{u^2}{v^2} \cdot u^3 v^3 \cdot \frac{2u}{v} \,du\,dv = \int_1^2 \left(\int_1^{\sqrt{2}} 2u^6 \,du \right) dv$

$= \frac{2\sqrt{2}^{\,7}}{7} - \frac{2}{7}.$

21]





$x+y=1$

$y=1-x$

$x+y+z=1$

$z=1-x-y$

S é o tetraedro

$$\int_S \overset{f(x,y,z)}{\overbrace{x+2y-z}}\, dxdydz = \int_B (f\circ T)(x,y,z)\cdot|\det T|\, dxdydz = \int_B \left[x-z+2(2x+y+z)-(3x+2y+z)\right]\cdot 2\, dxdydz$$

$$= \int_B 4x\, dxdydz = \int_0^1\int_0^{1-x}\int_0^{1-x-y} 4x\, dz\, dy\, dx$$

B: tetraedro de vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$.

$$= \int_0^1\int_0^{1-x} 4x(1-x-y)\, dy\, dx$$

$$= \int_0^1\int_0^{1-x} 4x+4x^2-4xy\, dy\, dx$$

$$= \int_0^1 4x(1-x)+4x(1-x)-2x(1-x)^2\, dx$$

$$= \dots$$

$T(1,0,0)=(1,2,3)$

$T(0,1,0)=(0,1,2)$

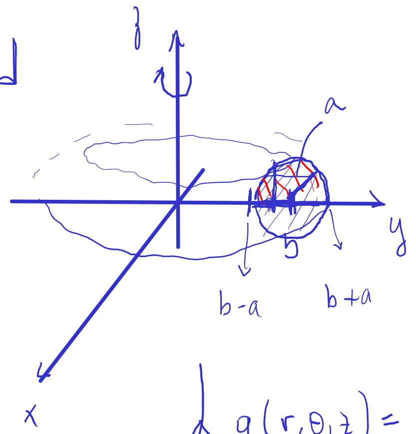
$T(0,0,1)=(-1,1,1)$

$T(x,y,z)=(x-z,\ 2x+y+z,\ 3x+2y+z)$

$$T' = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$\det T' = -2$

29



$$V = \int_R 1 \, dxdydz = 2 \cdot \int_0^{2\pi} \left(\int_{b-a}^{b+a} \left(\int_0^{\sqrt{a^2-(r-b)^2}} r \, . \, dz \right) dr \right) d\theta = 2\pi b . \pi a^2 .$$

$$(y-b)^2 + z^2 = a^2$$

$$g(r,\theta,z) = (r\cos\theta, r\sin\theta, z)$$

$$\det g' = r$$