$V = \mathbb{R}^3$

$\mathcal{B} = \{e_1, e_2, e_3\}$

$\mathcal{T}^1(V) = V^* = \{\varphi : \mathbb{R}^3 \to \mathbb{R},\ \varphi$ é linear$\}$

$\mathcal{B}^* = \{\varphi_1, \varphi_2, \varphi_3\}$

$$a^i e_i$$
$$\|$$
$$v = a_1 e_1 + a_2 e_2 + a_3 e_3$$

$$\left.\begin{aligned} \varphi_1(e_1) &= 1 \\ \varphi_1(e_2) &= 0 \\ \varphi_1(e_3) &= 0 \end{aligned}\right\} \quad \begin{aligned} \varphi_1(v) &= a_1 \\ \varphi_2(v) &= a_2 \\ \varphi_3(v) &= a_3 \end{aligned}$$

$$\mathcal{T}^2(\mathbb{R}^3) = \{T : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}\} \Rightarrow T = \sum_{i,j=1}^{3} a_{ij}\, \varphi_i \otimes \varphi_j$$

$$T(u,v) = \sum_{i,j=1}^{3} a_{ij}\, u_i v_j$$

$$(a_{ij}) = Id \Rightarrow T(u,v) = \langle u, v \rangle.$$

$$\langle u, v \rangle = \sum_{i=1}^{3} 1 \cdot \varphi_i \otimes \varphi_i$$

$$\varphi_i \otimes \varphi_j : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$(v,w) \longmapsto \varphi_i \otimes \varphi_j(v,w) = \varphi_i(v) \cdot \varphi_j(w) = v_i \cdot w_j$$

$\mathcal{B} = \{\varphi_{i_1} \otimes \dots \otimes \varphi_{i_k},\ 1 \leq i_1, \dots, i_k \leq n\}$ é base de $\mathcal{T}^k(V)$

$$\underline{n} \cdot \underline{n} \cdots \underline{n} = n^k$$

$f : V \to W$, $T \in \mathcal{T}^k(W)$ $\quad f^*T \in \mathcal{T}^k(V)$, $f^*T(v_1, \dots, v_k) = T(f(v_1), \dots, f(v_k))$

$S \in \mathcal{T}^k(W)$, $T \in \mathcal{T}^l(W)$

$$\begin{aligned} f^*(S \otimes T)(v_1, \dots, v_k, v_{k+1}, \dots, v_{k+l}) &= (S \otimes T)(f(v_1), \dots, f(v_{k+l})) \\ &= S(f(v_1), \dots, f(v_k)) \cdot T(f(v_{k+1}), \dots, f(v_{k+l})) \\ &= f^*S(v_1, \dots, v_k) \cdot f^*T(v_{k+1}, \dots, v_{k+l}) \\ &= (f^*S \otimes f^*T)(v_1, \dots, v_{k+l}). \end{aligned}$$

$V=\mathbb{R}^2$, $\mathcal{T}^2(\mathbb{R}^2) \ni T$, $\{\varphi_1,\varphi_2\}$ base dual de $\{e_1,e_2\}$

$T = \varphi_1\otimes\varphi_2 + \varphi_2\otimes\varphi_1$

$$Alt(T)(v_1,v_2) = \frac{1}{2!}\sum_{\sigma\in S_2} sgn(\sigma)\cdot T(v_{\sigma(1)}, v_{\sigma(2)})$$

$$= \frac{1}{2!}\left[1\cdot T(v_1,v_2) - 1\cdot T(v_2,v_1)\right]$$

$$= \frac{1}{2}\left[v_{11}v_{22}+v_{21}v_{12} - v_{12}v_{21} - v_{22}\cdot v_{11}\right]$$

$$=0$$

$\sigma_1 = id$: $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = (1)(2)$ é par

$\sigma_2 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = (1\ 2)$ é ímpar

$$T((v_{11},v_{21}),(v_{12},v_{22})) = \varphi_1(v_1)\cdot\varphi_2(v_2)+\varphi_2(v_1)\cdot\varphi_1(v_2)$$

$$= v_{11}v_{22}+v_{21}\cdot v_{12}$$

$$S=\varphi_1\otimes\varphi_2 \Rightarrow Alt(S) = \frac{1}{2}\cdot\left[v_{11}v_{22}-v_{12}v_{2}\right] = \frac{1}{2}\det\begin{vmatrix} v_{11} & v_{21} \\ v_{21} & v_{22}\end{vmatrix}$$

$Alt(S)$ é anti-simétrico