$g : \mathbb{R}^m \to \mathbb{R}$, $g \in \Lambda^0(\mathbb{R}^m)$ $\quad (f^* g)(p) = (g \circ f)(p)$

$\qquad\qquad\qquad\qquad\qquad\qquad =$

$f : \mathbb{R}^n \to \mathbb{R}^m$

$f_* : \mathbb{R}^n_p \to \mathbb{R}^m_{f(p)}$

$f^* g \in \Lambda^0(\mathbb{R}^n)$

$\omega \in \Lambda^1(\mathbb{R}^3) \Rightarrow \omega = P\,dx + Q\,dy + R\,dz$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ \partial x & \partial y & \partial z \\ P & Q & R \end{vmatrix} = \operatorname{rot}(P, Q, R)$$

$$d\omega = P_x\, dx \wedge dx + P_y\, dy \wedge dx + P_z\, dz \wedge dx +$$
$$Q_x\, dx \wedge dy + Q_y\, dy \wedge dy + Q_z\, dz \wedge dy$$
$$R_x\, dx \wedge dz + R_y\, dy \wedge dz + R_z\, dz \wedge dz$$
$$= (Q_x - P_y)\, dx \wedge dy + (R_x - P_z)\, dx \wedge dz + (R_y - Q_z)\, dy \wedge dz \in \Lambda^2(\mathbb{R}^3)$$

$$d^2\omega = d(d\omega) = (Q_{xz} - P_{yz})\, \overset{dx \wedge dy \wedge dz}{dz \wedge dx \wedge dy} +$$
$$(-R_{xy} \overset{+}{-} P_{zy})\, \overset{dx \wedge dy \wedge dz}{dy \wedge dx \wedge dz} +$$
$$(R_{yx} - Q_{zx})\, dx \wedge dy \wedge dz = (\quad 0 \quad)\, dx \wedge dy \wedge dz.$$

$$d^2\omega \equiv 0, \quad \Lambda^3(\mathbb{R}^3).$$

$$d\Big(\sum_{i_1<\dots<i_l} f\eta_{i_1\dots i_l}\cdot dx_{i_1}\wedge\dots\wedge dx_{i_l}\Big)=\sum_{i_1<\dots<i_l}\sum_{\alpha=1}^{n}\frac{\partial(f\cdot\eta_{i_1\dots i_l})}{\partial x^\alpha}\,dx^\alpha\wedge dx_{i_1}\wedge\dots\wedge dx_{i_l}$$

$$=\underbrace{\sum_{\alpha=1}^{n}\frac{\partial f}{\partial x_\alpha}dx_\alpha}_{df}\wedge\underbrace{\sum_{i_1<\dots<i_l}\eta_{i_1\dots i_l}\,dx_{i_1}\wedge\dots\wedge dx_{i_l}}_{\eta}+f\cdot\overbrace{\sum_{i_1<\dots<i_l}\frac{\partial\eta_{i_1\dots i_l}}{\partial x_\alpha}dx_\alpha\wedge dx_{i_1}\wedge\dots\wedge dx_{i_l}}^{d\eta}=df\wedge\eta+f\wedge d\eta.$$

$$\omega=\sum\omega_{i_1\dots i_k}\,dx_{i_1}\wedge\dots\wedge dx_{i_k}$$

$$\eta=\sum\eta_{i_1\dots i_l}\,dx_{i_1}\wedge\dots\wedge dx_{i_l}$$

$$\Rightarrow\ d(\omega\wedge\eta)=d\Big(\sum_I\sum_J\omega_I\eta_J\,dx_I\wedge dx_J\Big)$$

$$=\sum_{I,J}\sum_\alpha\frac{\partial(\omega_I\eta_J)}{\partial x_\alpha}dx_\alpha\wedge dx_I\wedge dx_J$$

$$=\sum_{I,J}\sum_\alpha\frac{\partial\omega_I}{\partial x_\alpha}dx_\alpha\wedge dx_I\wedge\eta_J\,dx_J+\sum_{I,J}\sum_\alpha\omega_I\frac{\partial\eta_J}{\partial x_\alpha}dx_\alpha\wedge dx_I\wedge dx_J$$

$$=d\omega\wedge\eta+\omega\wedge(-1)^k d\eta=d\omega\wedge\eta+(-1)^k\omega\wedge d\eta.$$