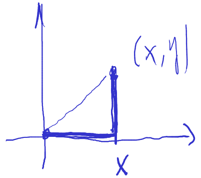
$f: \mathbb{R}^2 \to \mathbb{R}$, tq $df = P\,dx + Q\,dy = \omega$

$$f(x,y) = \int_0^x P(t,0)\,dt + \int_0^y Q(x,t)\,dt$$

$$\frac{\partial f}{\partial x} = P(x,0) + \int_0^y \underbrace{Q_x}_{P_y}(x,t)\,dt$$

$$= P(x,0) + P(x,y) - P(x,0) = P(x,y).$$

$$\frac{\partial f}{\partial y} = Q(x,y). \Rightarrow \omega = df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy.$$



$d\omega = 0 \Leftrightarrow Q_x = P_y$

$\omega \in \Lambda^1(A)$, $A \subset \mathbb{R}^n$, aberto estrelado em 0. $\omega = \sum \omega_i\,dx_i$

$$d\omega = \sum_{1 \le i,j \le n} \frac{\partial \omega_i}{\partial x_j}\,dx_j \wedge dx_i$$ é fechada se e só se $\Rightarrow \frac{\partial \omega_i}{\partial x_j} = \frac{\partial \omega_j}{\partial x_i}$

$$= \sum_{1 \le i \le j \le n} \frac{\partial \omega_i}{\partial x_j} - \frac{\partial \omega_j}{\partial x_i}\,dx_j \wedge dx_i$$

$$I\omega = \sum_{i=1}^n \int_0^1 \omega_i(tx)\,x_i\,dt$$

$$d(I\omega) = \int_0^1 \sum_{j=1}^n \left(\sum_{i=1}^n x_i \frac{\partial \omega_i}{\partial x_j}(tx).t + \omega_i(tx)\frac{\partial x_i}{\partial x_j} \right) dx_j\,dt$$

$$= \int_0^1 \left(\sum_{j=1}^n \omega_j(tx)\,dx_j + t \sum_{i,j=1}^n x_i \frac{\partial \omega_j}{\partial x_i}(tx)\,dx_j \right) dt$$

$$= \int_0^1 \sum_{j=1}^n \omega_j(tx)\,dx_j + \sum_{i,j=1}^n t\,x_i \frac{\partial \omega_j}{\partial x_i}(tx)\,dx_j)\,dt$$

$$= \int_0^1 \sum_{j=1}^n \frac{d}{dt}\left(t\,\omega_j(tx)\right) dx_j\,dt$$

$$= \sum_{j=1}^n \int_0^1 \frac{d}{dt}\left(t\,\omega_j(tx)\right) dt\,dx_j = \sum_{j=1}^n \omega_j(x)\,dx_j =: \omega.$$