

$$7. c) \sum_{i=1}^n \frac{1}{i^2} \leq 2 - \frac{1}{n}$$

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$$\sum_{i=1}^{k+1} \frac{1}{i^2} = \sum_{i=1}^k \frac{1}{i^2} + \frac{1}{(k+1)^2}$$

$$\leq 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$$

$$= 2 - \left[\frac{1}{k} - \frac{1}{(k+1)^2} \right]$$

$$= 2 - \left[\frac{(k+1)^2 - k}{k(k+1)^2} \right]$$

$$= 2 - \left[\frac{k^2 + 2k + 1 - k}{k(k+1)^2} \right]$$

$$= 2 - \frac{k^2 + k}{k(k+1)^2} - \frac{1}{k(k+1)^2}$$

$$= 2 - \frac{k(k+1)}{k(k+1)^2} - \frac{1}{k(k+1)^2}$$

$$= 2 - \frac{1}{k+1} - \frac{1}{k(k+1)^2}$$

$$< 2 - \frac{1}{k+1}$$

Note que como $k \geq 1$

$$-\frac{1}{k+1} > -\frac{1}{k+1} - \frac{1}{k(k+1)^2}$$

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