On the base problem for the polynomial identities of matrix algebras*

Alexey Kuz'min

Since 1950, the Amitsur-Levitzki Theorem [1]–[7] is known: the minimal polynomial identity for the algebra $M_n(F)$ of $n \times n$ matrices over a field F of characteristic 0 is the *standard polynomial*

$$\mathrm{st}_{2n} = \sum_{\sigma \in \mathrm{S}_{2n}} \mathrm{sgn}(\sigma)\, x_{\sigma(1)} x_{\sigma(2)} \ldots x_{\sigma(2n)},$$

where S_k is the group of permutations on the elements $\{1, 2, \ldots, k\}$ and $\mathrm{sgn}(\sigma) = \begin{cases} 1, & \text{if } \sigma \text{ is even,} \\ -1, & \text{if } \sigma \text{ is odd.} \end{cases}$ In 1973, Ju. P. Razmyslov [8] provided the 9 polynomials forming a base for identities of $M_2(F)$. V. S. Drensky [9] reduced the Razmyslov's base up to 2 polynomials. However, the problem on finding a base for polynomial identities of $M_n(F)\,(n \geqslant 3)$ is still open and there is no solution in sight, even for $n = 3$. Actually, we only know [10, 11] that all the identities of $M_3(F)$ of degree $d \leqslant 8$ follow from st_6.

In this talk, we consider an embedding of $M_n(F)$ into a $\mathbb{Z}_n$-*graded associative ring* $A^{(n)}$ obtained as a *quotient ring* $A^{(n)} = R[x, \varphi]/(x^n - 1)$ for a *ring* $R[x, \varphi]$ *of* (associative but not commutative) *skew-polynomials* on one variable x over a free associative commutative ring R with an automorphism $\varphi : R \mapsto R$. We present some results on the connections between the polynomial identities of $M_n(F)$ and $A^{(n)}$. We also formulate some problems on the identities of $A^{(n)}$ related naturally with $M_n(F)$.

References

[1] A. S. Amitsur, J. Levitzki, Minimal identities for algebras, *Proc. Am. Math. Soc.* 1 (1950), 449–463.

*Supported by the Capes–PPgMAE program of the Federal University of Rio Grande do Norte (UFRN)

[2] B. Kostant, A theorem of Frobenius, a theorem of Amitsur-Levitski and cohomology theory, *J. Math. Mech.* 7 (1958), 237–264.

[3] R. G. Swan, An application of graph theory to algebra, *Proc. Am. Math. Soc.* 14 (1963), 367–373.

[4] Ju. P. Razmyslov, Identities with trace in full matrix algebras over a field of characteristic zero, *Math. USSR, Izv.* 8 (1975), 727–760.

[5] S. Rosset, A new proof of the Amitsur-Levitski identity, *Isr. J. Math.* 23 (1976), 187–188.

[6] J. Szigeti, Z. Tuza, G. Revesz, Eulerian polynomial identiies on matrix rings, *J. Algebra* 161 (1993), 90–101.

[7] C. Procesi, On the theorem of Amitsur-Levitski, *Isr. J. Math.* 207 (2015), 151–154.

[8] Ju. P. Razmyslov, Finite basing of the identities of a matrix algebra of second order over a field of characteristic 0, *Algebra Logic* 12 (1974), 47–63.

[9] V. S. Drensky, A minimal basis of identities for a second-order matrix algebra over a field of characteristic 0, *Algebra Logic* 20 (1982), 188–194.

[10] V. S. Drensky, A. Kasparian, Polynomial identities of eighth degree for 3×3 matrices, *God. Sofij. Univ., Fak. Mat. Mekh.* 77 (1988), 175–195.

[11] S. Bondari, Constructing the polynomial identities and central identities of degree < 9 of 3×3 matrices, *Linear Algebra Appl.* 258 (1997), 233–249.