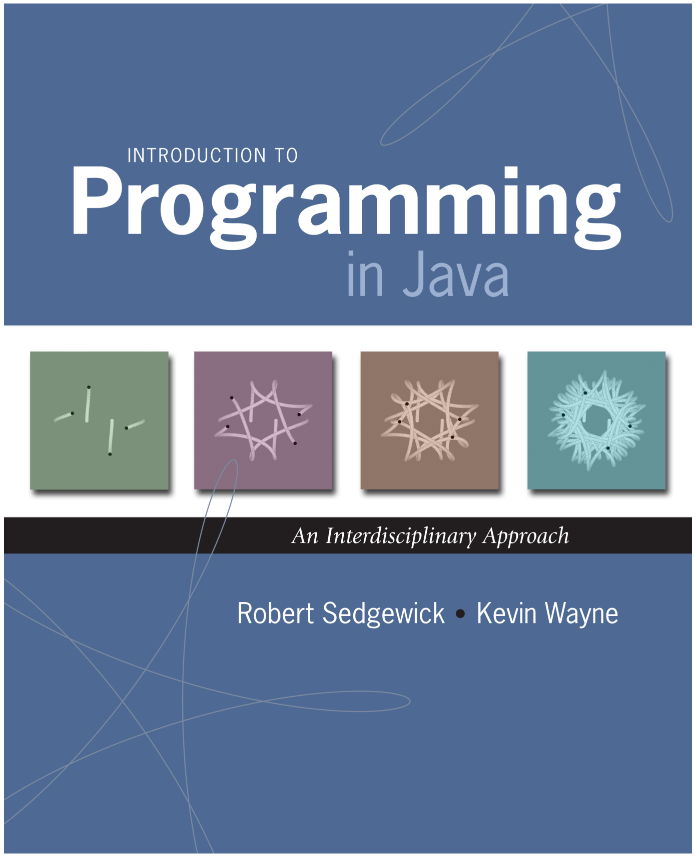
4.4 Symbol Tables



Symbol Table

Symbol table. Key-value pair abstraction.

- Insert a key with specified value.
- Given a key, search for the corresponding value.

Ex. [DNS lookup]

- Insert URL with specified IP address.
- Given URL, find corresponding IP address.

URL	IP address
www.cs.princeton.edu	128.112.136.11
www.princeton.edu	128.112.128.15
www.yale.edu	130.132.143.21
www.harvard.edu	128.103.060.55
www.simpsons.com	209.052.165.60

key value

Symbol Table Applications

Application	Purpose	Key	Value
phone book	look up phone number	name	phone number
bank	process transaction	account number	transaction details
file share	find song to download	name of song	computer ID
file system	find file on disk	filename	location on disk
dictionary	look up word	word	definition
web search	find relevant documents	keyword	list of documents
book index	find relevant pages	keyword	list of pages
web cache	download	filename	file contents
genomics	find markers	DNA string	known positions
DNS	find IP address given URL	URL	IP address
reverse DNS	find URL given IP address	IP address	URL
compiler	find properties of variable	variable name	value and type
routing table	route Internet packets	destination	best route

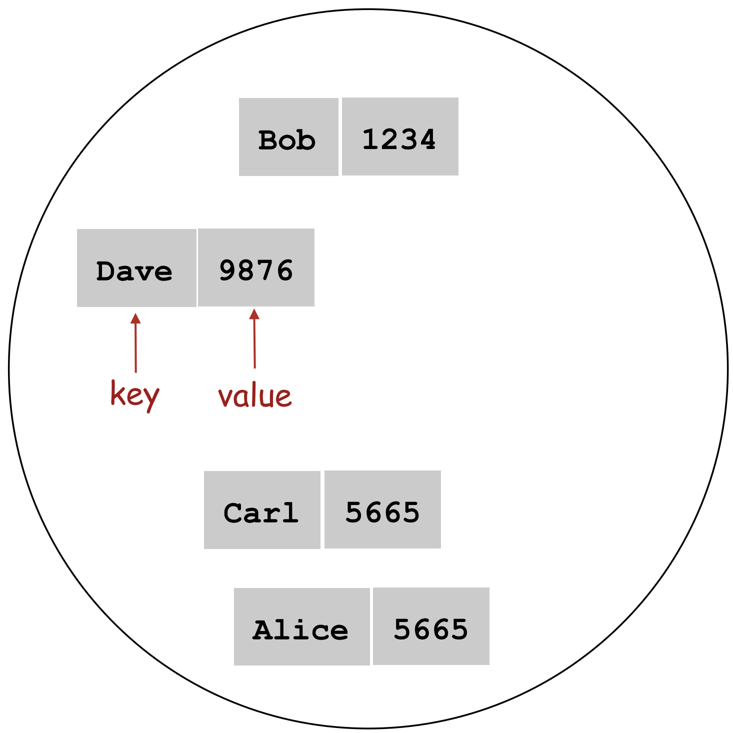
Symbol Table API

`public class *ST<Key extends Comparable<Key>, Value>`

	`*ST()`	*create a symbol table*
`void`	`put(Key key, Value v)`	*put* `key-value` *pair into the table*
`Value`	`get(Key key)`	*return value paired with* `key`, `null` *if* `key` *not in table*
`boolean`	`contains(Key key)`	*is there a value paired with* `key`*?*

Note: Implementations should also implement the `Iterable<Key>` *interface to enable clients to access keys in sorted order with foreach loops.*

symbol table stores a set of key-value pairs

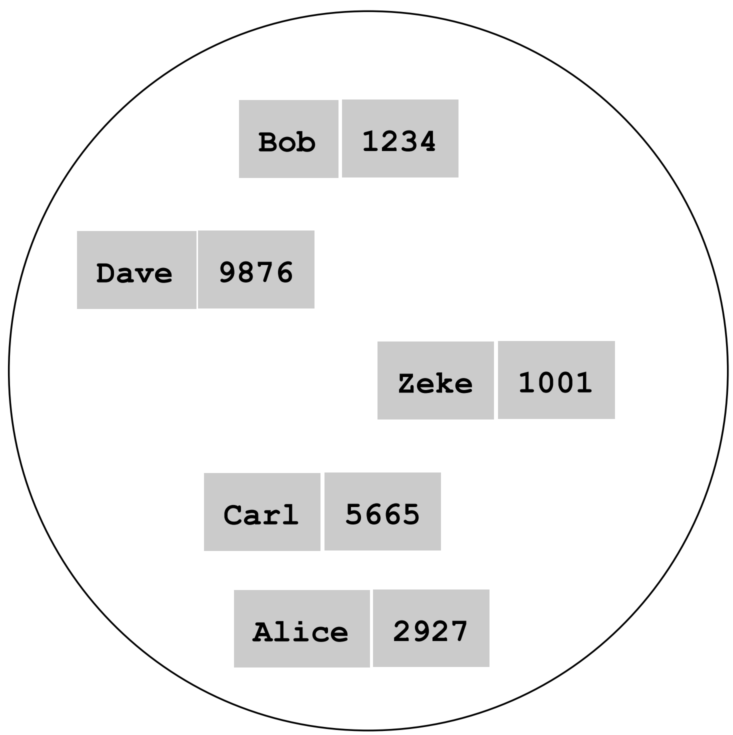


Symbol Table API

public class *ST<Key extends Comparable<Key>, Value>		
	*ST()	*create a symbol table*
void	put(Key key, Value v)	*put* key-value *pair into the table*
Value	get(Key key)	*return value paired with* key, null *if* key *not in table*
boolean	contains(Key key)	*is there a value paired with* key?

Note: Implementations should also implement the Iterable<Key> *interface to enable clients to access keys in sorted order with foreach loops.*

put("Zeke", 1001)
adds key-value pair



Symbol Table API

public class *ST<Key extends Comparable<Key>, Value>		
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void	put(Key key, Value v)	*put* key-value *pair into the table*
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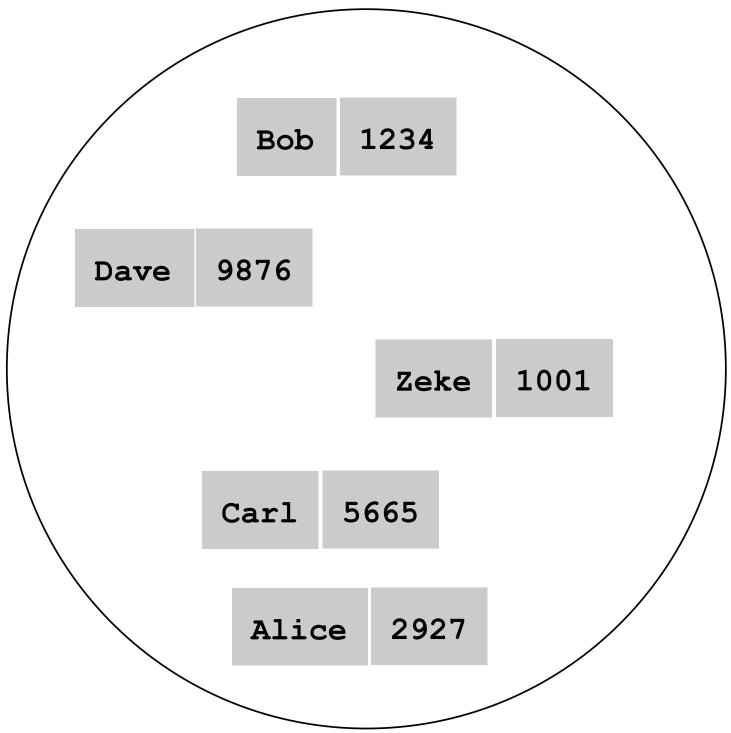
boolean	contains(Key key)	*is there a value paired with* key?

Note: Implementations should also implement the Iterable<Key> *interface to enable clients to access keys in sorted order with foreach loops.*

get("Alice")
returns 5665



Symbol Table API

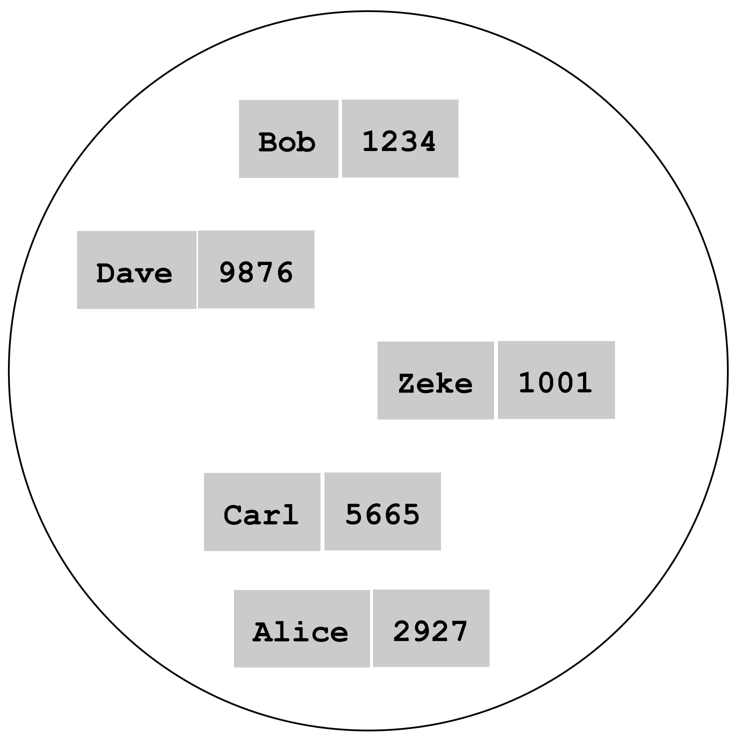
```
public class *ST<Key extends Comparable<Key>, Value>
```

	*ST()	*create a symbol table*
void	put(Key key, Value v)	*put* key-value *pair into the table*
Value	get(Key key)	*return value paired with* key, null *if* key *not in table*
boolean	contains(Key key)	*is there a value paired with* key?

Note: Implementations should also implement the Iterable<Key> *interface to enable clients to access keys in sorted order with foreach loops.*

contains("Alice")
returns true

contains("Kevin")
returns false



Symbol Table API

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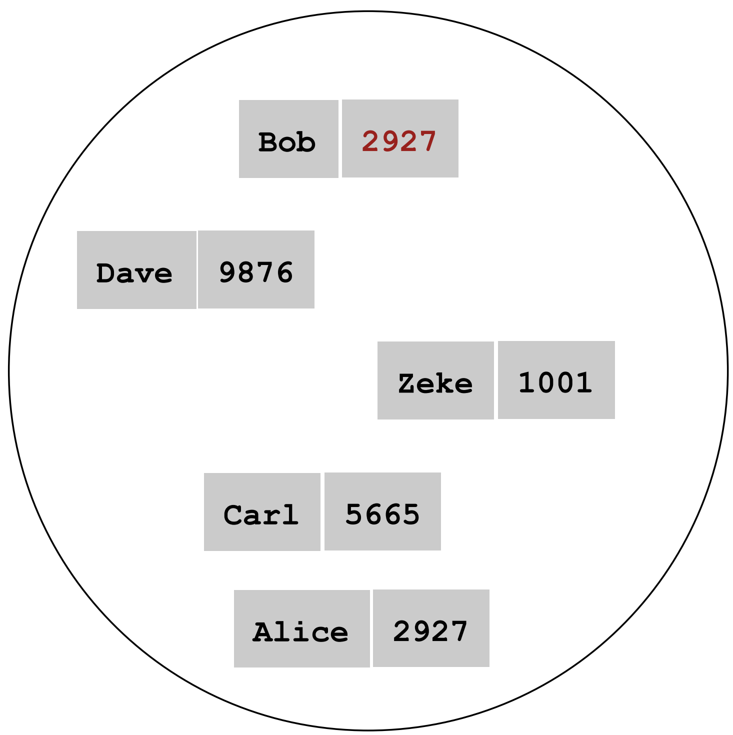
`boolean`	`contains(Key key)`	*is there a value paired with* `key`*?*

Note: Implementations should also implement the `Iterable<Key>` *interface to enable clients to access keys in sorted order with foreach loops.*

put(“Bob”, 2927)
changes Bob’s value

associative array notation
st[“Bob”] = 2927
is legal in some languages
(not Java)



Symbol Table Client: Frequency Counter

Frequency counter. [e.g., web traffic analysis, linguistic analysis]

- Read in a key.
- If key is in symbol table, increment count by one;
  If key is not in symbol table, insert it with count = 1.

key type   value type

```
public class Freq {
   public static void main(String[] args) {
      ST<String, Integer> st = new ST<String, Integer>();
      while (!StdIn.isEmpty()) {
         String key = StdIn.readString();
         if (st.contains(key)) st.put(key, st.get(key) + 1);
         else                  st.put(key, 1);
      }
```

calculate frequencies

enhanced for loop (stay tuned)

```
      for (String s : st)
         StdOut.println(st.get(s) + " " + s);
   }
}
```

print results

Sample Datasets

Linguistic analysis. Compute word frequencies in a piece of text.

File	Description	Words	Distinct
`mobydick.txt`	Melville's Moby Dick	210,028	16,834
`leipzig100k.txt`	100K random sentences	2,121,054	144,256
`leipzig200k.txt`	200K random sentences	4,238,435	215,515
`leipzig1m.txt`	1M random sentences	21,191,455	534,580

Reference: Wortschatz corpus, Univesität Leipzig

`http://corpora.informatik.uni-leipzig.de`

Zipf's Law

Linguistic analysis. Compute word frequencies in a piece of text.

```
% java Freq < mobydick.txt
4583 a
2 aback
2 abaft
3 abandon
7 abandoned
1 abandonedly
2 abandonment
2 abased
1 abasement
2 abashed
1 abate
…
```

```
% java Freq < mobydick.txt | sort -rn
13967 the
6415 of
6247 and
4583 a
4508 to
4037 in
2911 that
2481 his
2370 it
1940 i
1793 but
…
```

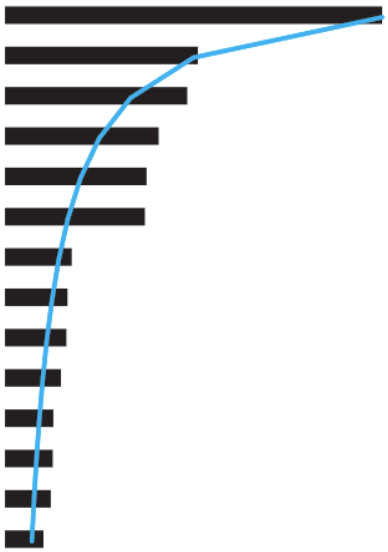
Zipf's law. In natural language, frequency of i^{th} most common word is inversely proportional to i.

e.g., most frequent word occurs about twice as often as second most frequent one

Zipf's Law

Linguistic analysis. Compute word frequencies in a piece of text.

```
% java Freq < leipzig1m.txt | sort -rn
1160105 the
593492 of
560945 to
472819 a
435866 and
430484 in
205531 for
192296 The
188971 that
172225 is
148915 said
147024 on
141178 was
118429 by
…
```



Zipf's law. In natural language, frequency of i^{th} most common word is inversely proportional to i.

e.g., most frequent word occurs about twice as often as second most frequent one

Symbol Table: Elementary Implementations

Unordered array.

- Put: add key to the end (if not already there).
- Get: scan through all keys to find desired value.

32	26	47	82	4	20	58	56	14	6	55		

Ordered array.

- Put: find insertion point, and shift all larger keys right.
- Get: binary search to find desired key.

4	6	14	20	26	32	47	55	56	58	82		
4	6	14	20	26	28	32	47	55	56	58	82	

insert 28

Binary Search: Mathematical Analysis

Analysis. To binary search in an array of size N: do one compare, then binary search in an array of size $N / 2$.

$$N \to N/2 \to N/4 \to N/8 \to \ldots \to 1$$

Q. How many times can you divide a number by 2 until you reach 1?

A. $\log_2 N$.

$$
\begin{gathered}
1 \\
2 \to 1 \\
4 \to 2 \to 1 \\
8 \to 4 \to 2 \to 1 \\
16 \to 8 \to 4 \to 2 \to 1 \\
32 \to 16 \to 8 \to 4 \to 2 \to 1 \\
64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1 \\
128 \to 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1 \\
256 \to 128 \to 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1 \\
512 \to 256 \to 128 \to 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1 \\
1024 \to 512 \to 256 \to 128 \to 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1
\end{gathered}
$$

Symbol Table: Implementations Cost Summary

Unordered array. Hopelessly slow for large inputs.

Ordered array. Acceptable if many more searches than inserts; too slow if many inserts.

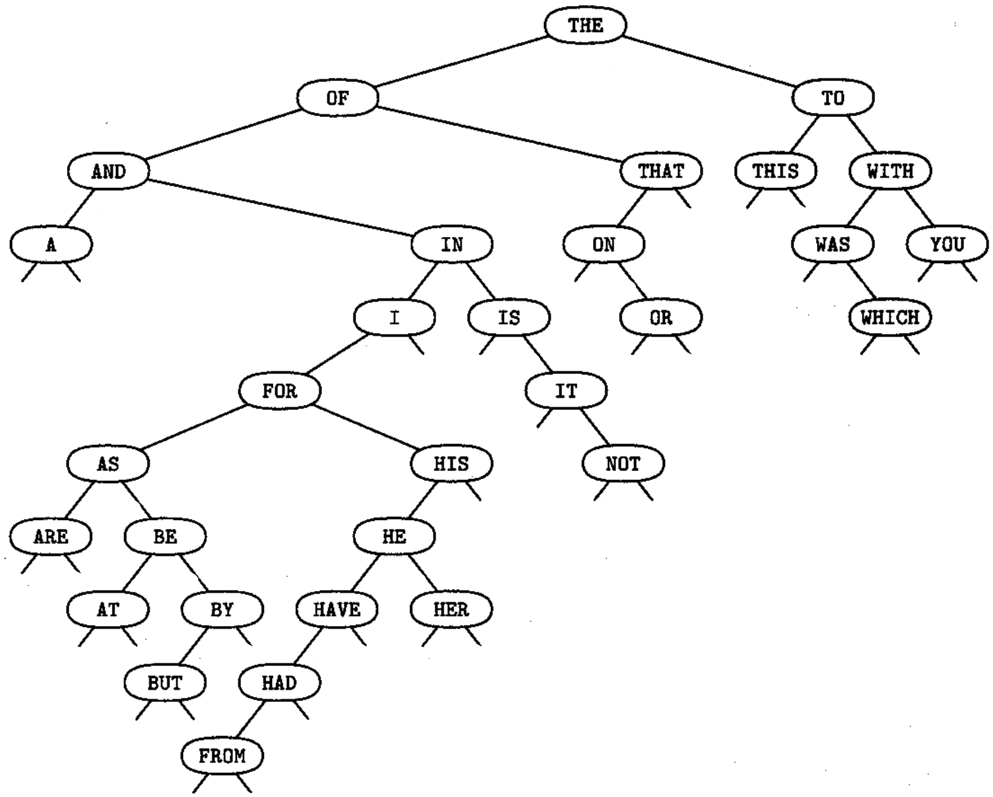
	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr

too slow (N^2 to build table)

doubling test (quadratic)

Challenge. Make all ops logarithmic.

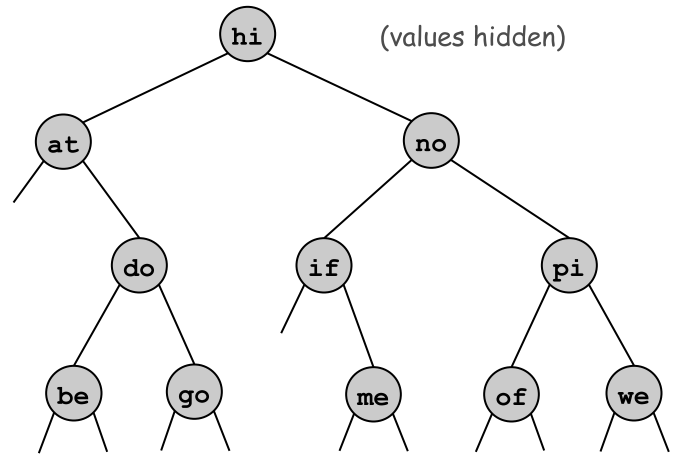
Binary Search Trees



Reference: Knuth, The Art of Computer Programming

Binary Search Trees

Def. A binary search tree is a binary tree in symmetric order.



Binary tree is either:

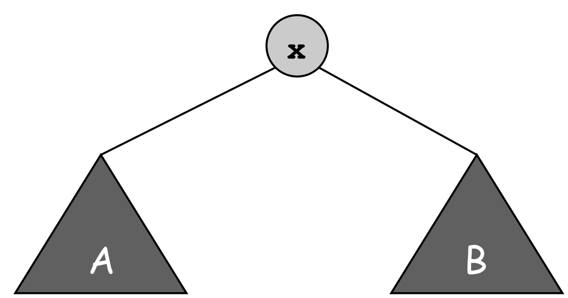
- Empty.
- A key-value pair and two binary trees.

we suppress values from figures

Symmetric order.

- Keys in left subtree are smaller than parent.
- Keys in right subtree are larger than parent.

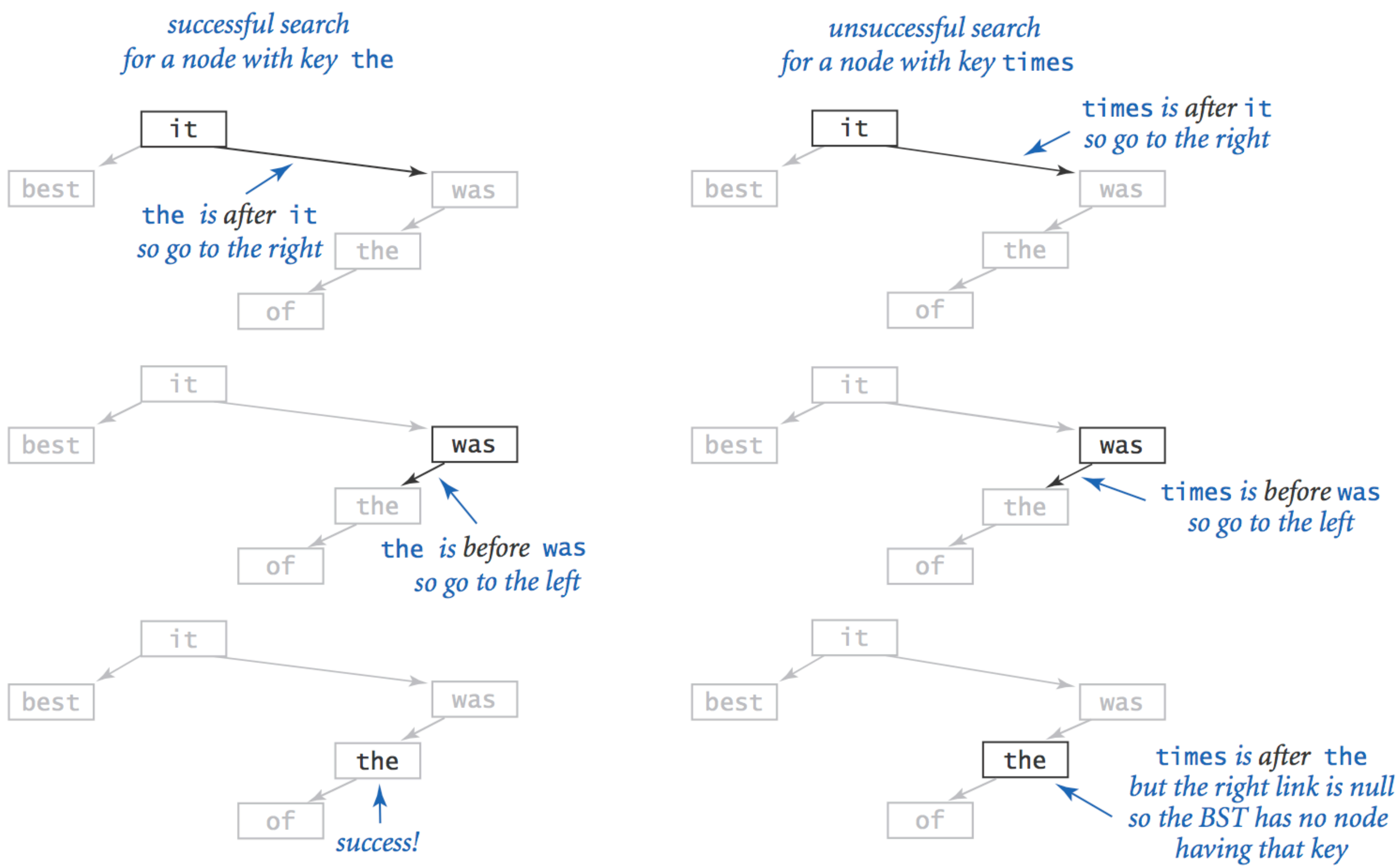
node



smaller keys

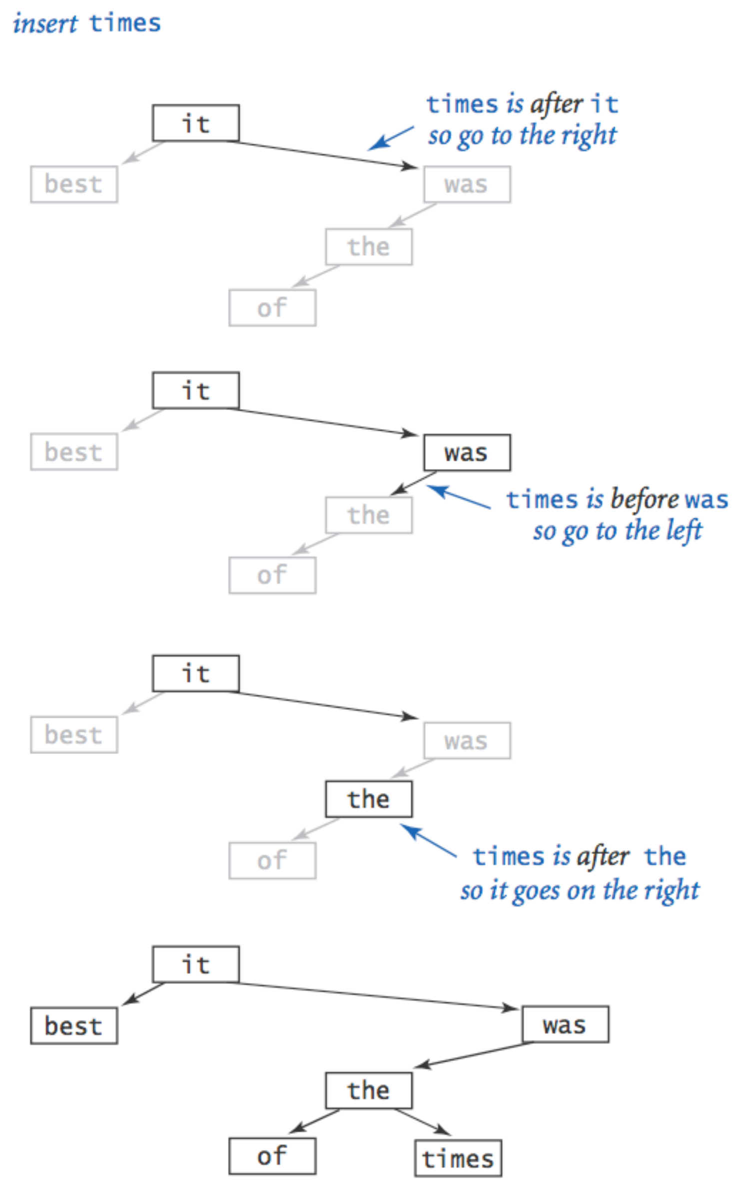
larger keys

BST Search



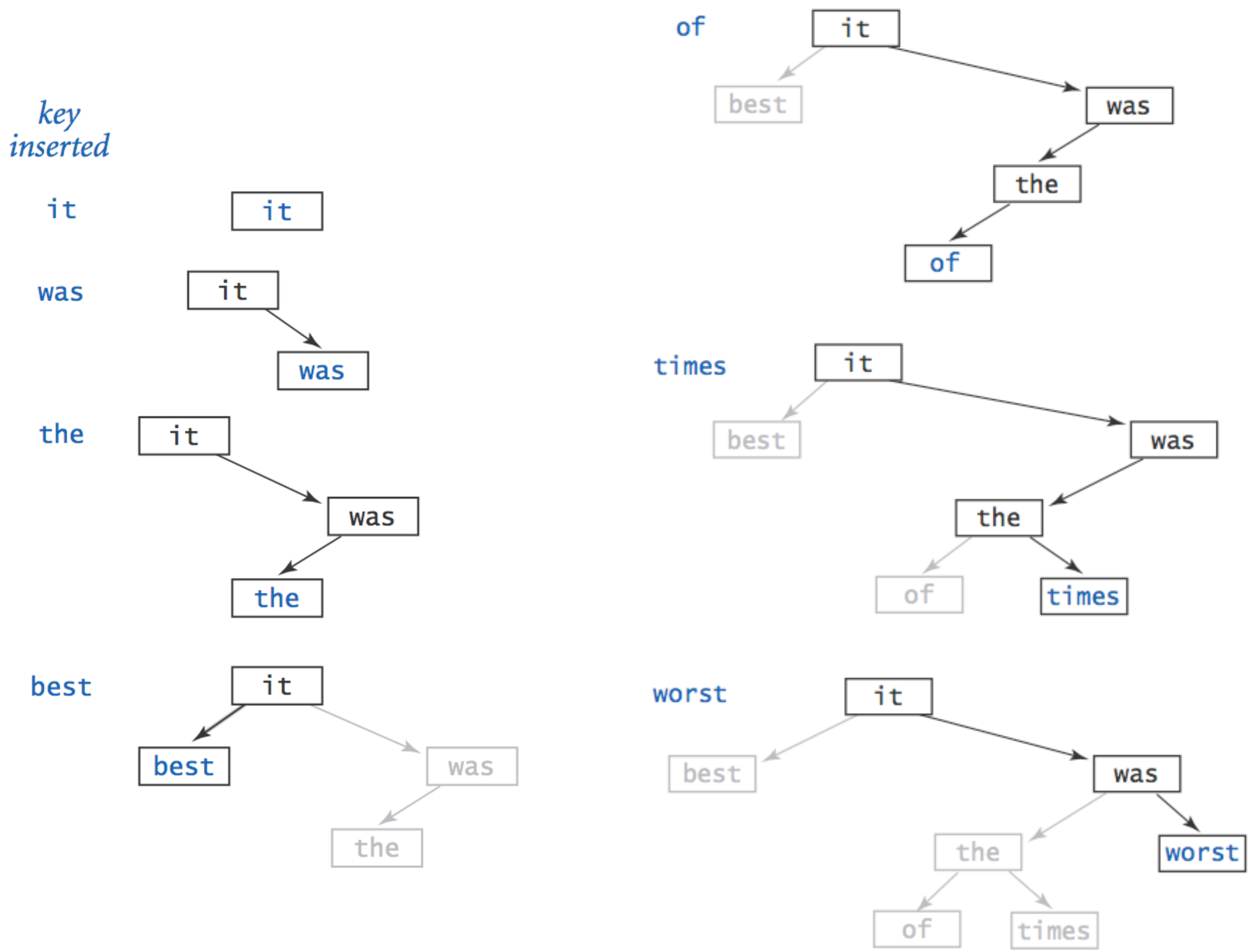
successful search
for a node with key the
it
best
was
the
of
the is after it
so go to the right
the is before was
so go to the left
success!
unsuccessful search
for a node with key times
times is after it
so go to the right
times is before was
so go to the left
times is after the
but the right link is null
so the BST has no node
having that key

BST Insert



insert times
it
best
was
the
of
times *is after* it
so go to the right
it
best
was
the
of
times *is before* was
so go to the left
it
best
was
the
of
times *is after* the
so it goes on the right
it
best
was
the
of
times

BST Construction



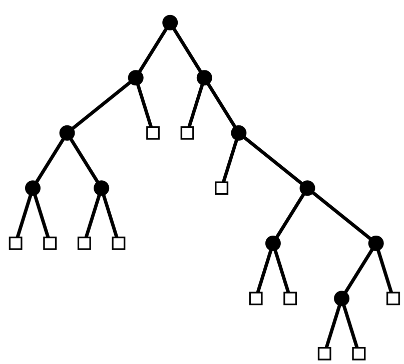
Binary Search Tree: Java Implementation

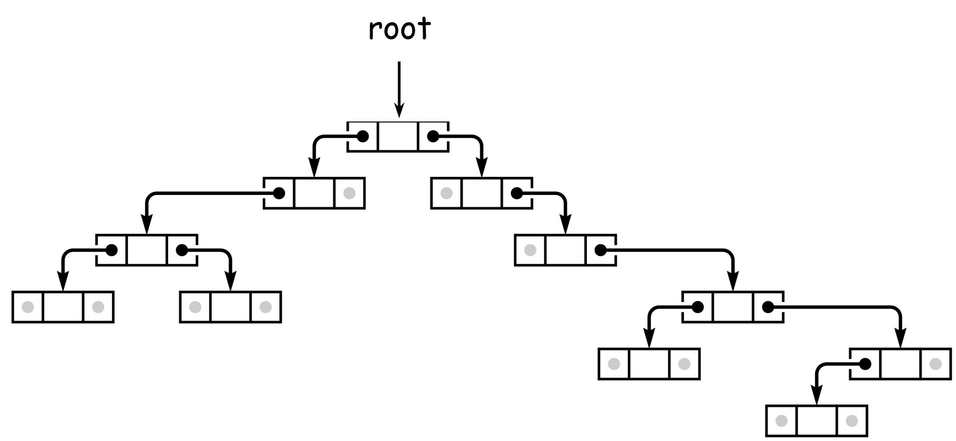
To implement: use two links per **Node**.

A **Node** is comprised of:

- A key.
- A value.
- A reference to the left subtree.
- A reference to the right subtree.

```
private class Node {
   private Key key;
   private Value val;
   private Node left;
   private Node right;
}
```





BST: Skeleton

BST. Allow generic keys and values.

requires `Key` to provide `compareTo()` method;
see book for details

```java
public class BST<Key extends Comparable<Key>, Value> {
   private Node root;   // root of the BST

   private class Node {
      private Key key;
      private Value val;
      private Node left, right;

      private Node(Key key, Value val) {
         this.key = key;
         this.val = val;
      }
   }

   public void put(Key key, Value val) { … }
   public Value get(Key key)           { … }
   public boolean contains(Key key)    { … }

}
```

BST: Get

Get. Return `val` corresponding to given `key`, or `null` if no such key.

```
public Value get(Key key) {
   return get(root, key);
}

private Value get(Node x, Key key) {
   if (x == null) return null;
   int cmp = key.compareTo(x.key);
   if      (cmp < 0) return get(x.left,  key);
   else if (cmp > 0) return get(x.right, key);
   else if (cmp > 0) return x.val;
}

public boolean contains(Key key) {
   return (get(key) != null);
}
```

negative if less,
zero if equal,
positive if greater

BST: Put

Put. Associate **val** with **key**.

- Search, then insert.
- Concise (but tricky) recursive code.

```
public void put(Key key, Value val) {
   root = put(root, key, val);
}

private Node put(Node x, Key key, Value val) {
   if (x == null) return new Node(key, val);
   int cmp = key.compareTo(x.key);
   if      (cmp < 0) x.left  = put(x.left,  key, val);
   else if (cmp > 0) x.right = put(x.right, key, val);
   else x.val = val;
   return x;
}
```

overwrite old value with new value

BST Implementation: Practice

Bottom line. Difference between a practical solution and no solution.

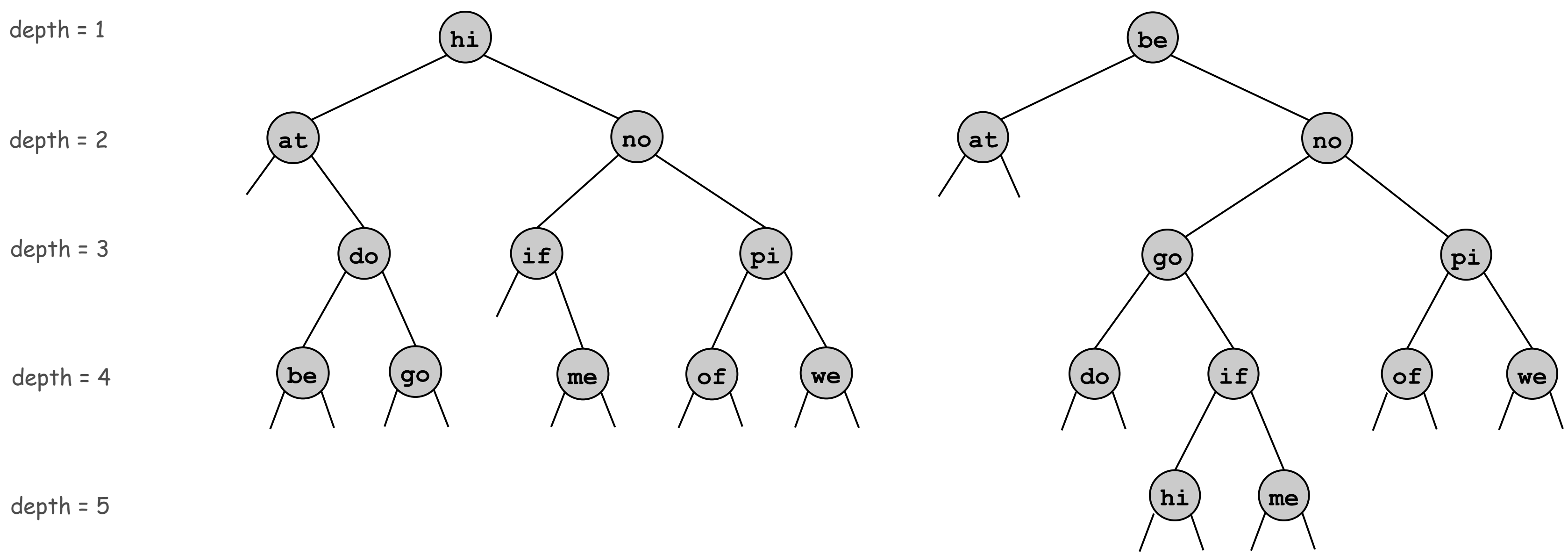
	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr
BST	?	?	.95 sec	7.1 sec	14 sec	69 sec

BST: Analysis

Running time per put/get.

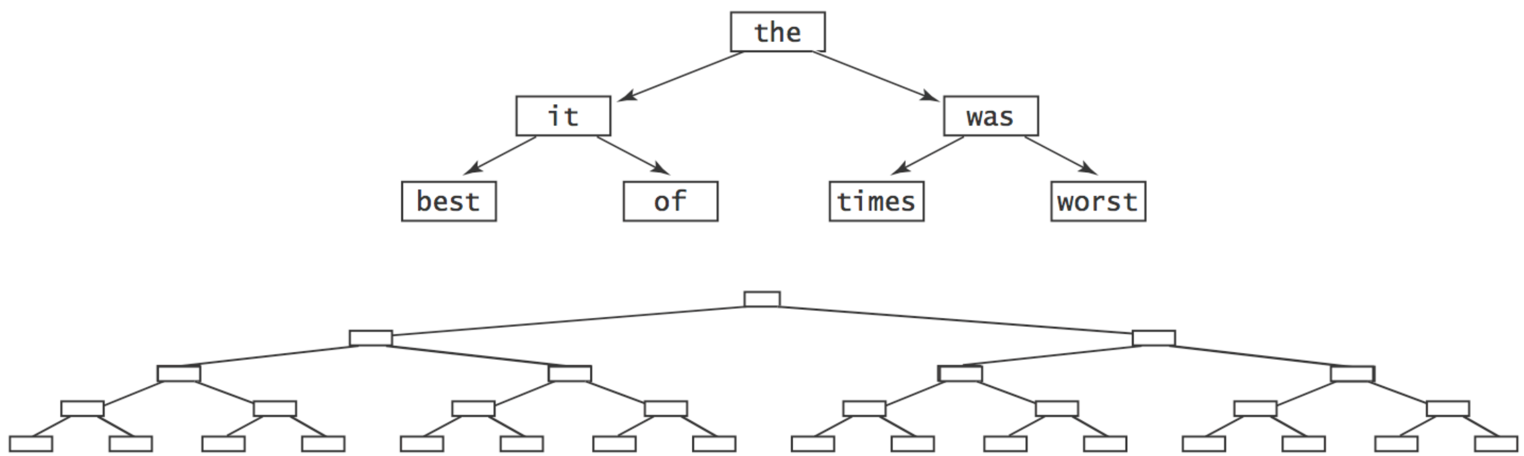
- There are many BSTs that correspond to same set of keys.
- Cost is proportional to depth of node.

number of nodes on path from root to node



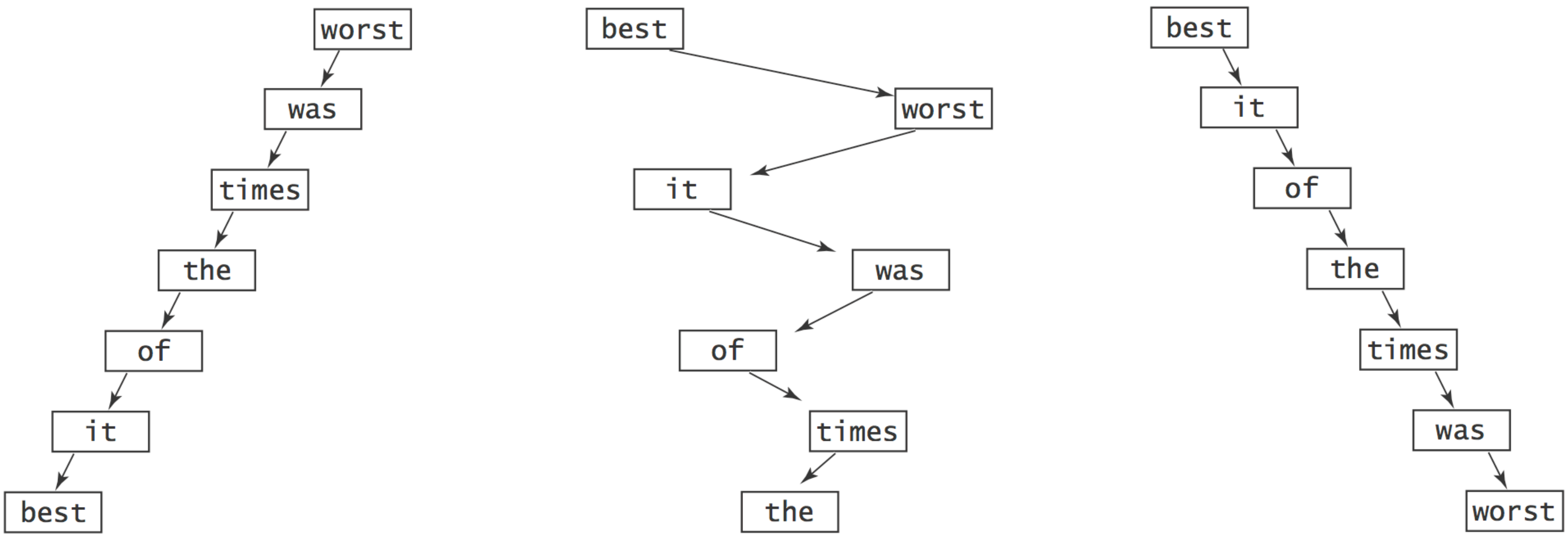
BST: Analysis

Best case. If tree is perfectly balanced, depth is at most $\lg N$.



BST: Analysis

Worst case. If tree is unbalanced, depth is N.



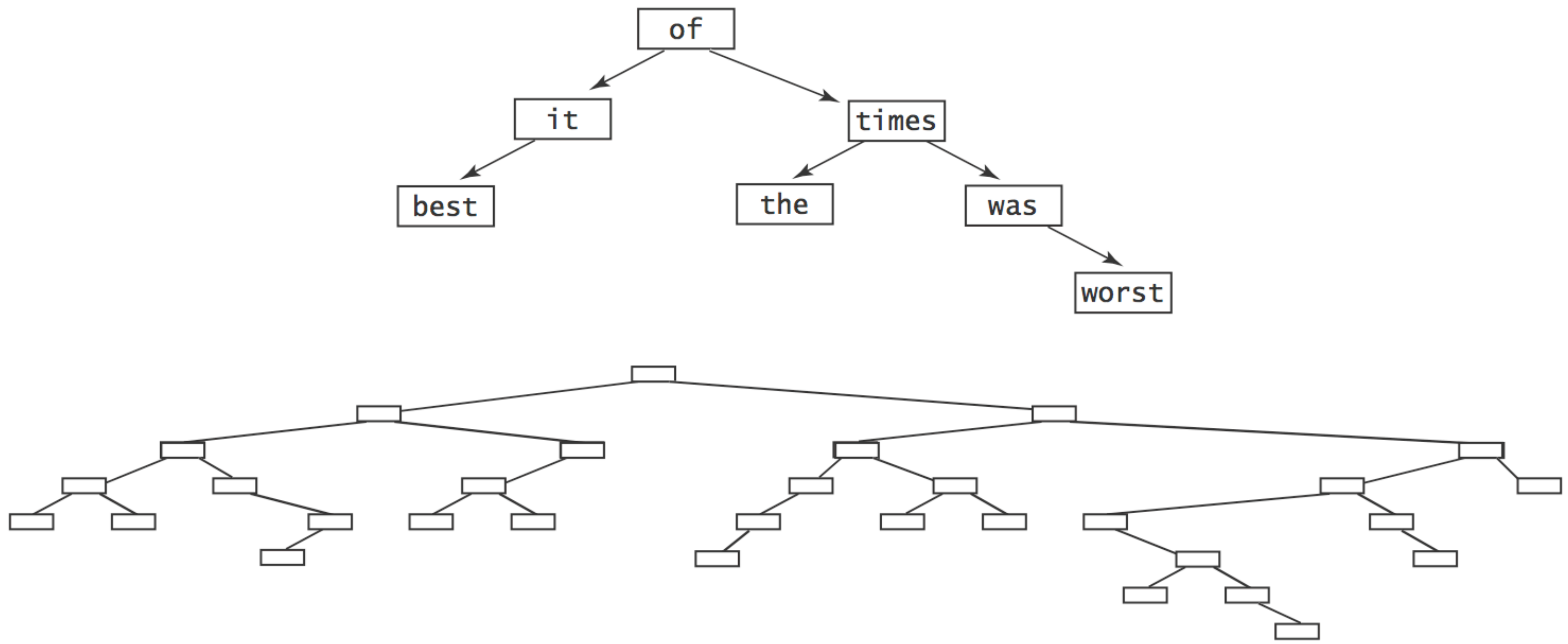
BST: Analysis

Average case. If keys are inserted in random order, trees stay flat.

BST: Analysis

Average case. If keys are inserted in random order, average depth is $2 \ln N$.

requires proof (see COS 226)



Typical BSTs constructed from randomly ordered keys

Symbol Table: Implementations Cost Summary

BST. Logarithmic time ops if keys inserted in random order.

	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr
BST	$\log N$ †	$\log N$ †	.95 sec	7.1 sec	14 sec	69 sec

† assumes keys inserted in random order

Q. Can we guarantee logarithmic performance?

Red-Black Tree

Red-black tree. A clever BST variant that guarantees depth $\leq 2 \lg N$.

see COS 226

Java red-black tree library implementation

```
import java.util.TreeMap;
import java.util.Iterator;

public class ST<Key extends Comparable<Key>, Value> implements Iterable<Key> {
   private TreeMap<Key, Value> st = new TreeMap<Key, Val>();

   public void put(Key key, Value val) {
      if (val == null) st.remove(key);
      else             st.put(key, val);
   }
   public Value get(Key key)         { return st.get(key);             }
   public Value remove(Key key)      { return st.remove(key);          }
   public boolean contains(Key key) { return st.containsKey(key);     }
   public Iterator<Key> iterator()  { return st.keySet().iterator(); }
}
```

Red-Black Tree

Red-black tree. A clever BST variant that guarantees depth $\leq 2 \lg N$.

see COS 226

	Running Time		Frequency Count			
implementation	get	put	Moby	100K	200K	1M
unordered array	N	N	170 sec	4.1 hr	-	-
ordered array	$\log N$	N	5.8 sec	5.8 min	15 min	2.1 hr
BST	$\log N$ †	$\log N$ †	.95 sec	7.1 sec	14 sec	69 sec
red-black	$\log N$	$\log N$	.95 sec	7.0 sec	14 sec	74 sec

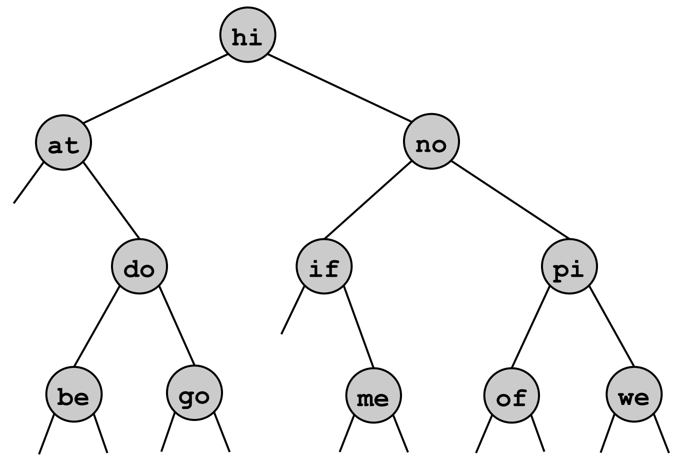
† assumes keys inserted in random order

Iteration

Inorder Traversal

Inorder traversal.

- Recursively visit left subtree.
- Visit node.
- Recursively visit right subtree.



`inorder: at be do go hi if me no of pi we`

```
public inorder() { inorder(root); }

private void inorder(Node x) {
   if (x == null) return;
   inorder(x.left);
   StdOut.println(x.key);
   inorder(x.right);
}
```

Enhanced For Loop

Enhanced for loop. Enable client to iterate over items in a collection.

```
ST<String, Integer> st = new ST<String, Integer>();
…

for (String s : st) {
   StdOut.println(st.get(s) + " " + s);
}
```

Enhanced For Loop with BST

BST. Add following code to support enhanced for loop.

see COS 226 for details

```
import java.util.Iterator;
import java.util.NoSuchElementException;

public class BST<Key extends Comparable<Key>, Value> implements Iterable<Key>  {

   private Node root;

   private class Node { … }

   public void put(Key key, Value val) { … }
   public Value get(Key key)           { … }
   public boolean contains(Key key)    { … }

   public Iterator<Key> iterator() { return new Inorder(); }
   private class Inorder implements Iterator<Key> {
       Inorder() { pushLeft(root); }
       public void remove()     { throw new UnsupportedOperationException(); }
       public boolean hasNext() { return !stack.isEmpty();                   }
       public Key next() {
          if (!hasNext()) throw new NoSuchElementException();
          Node x = stack.pop();
          pushLeft(x.right);
          return x.key;
       }
       public void pushLeft(Node x) {
          while (x != null) {
             stack.push(x);
             x = x.left;
          }
       }
   }
}
```

Symbol Table: Summary

Symbol table. Quintessential database lookup data type.

Choices. Ordered array, unordered array, BST, red-black, hash, ….

- Different performance characteristics.
- Java libraries: **`TreeMap`**, **`HashMap`**.

Remark. Better symbol table implementation improves all clients.