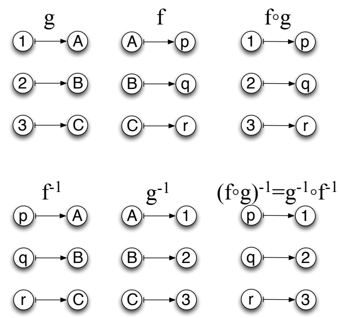
Errata for Edition 1 of *Coding the Matrix*, January 13, 2017

Your copy might not contain some of these errors. Most do not occur in the copies currently being sold as April 2015.

- Section 0.3: "... the input is a *pre-image* of the input" should be "... the input is a *pre-image* of the output".
- Figure 4 in Section 0.3.8: The figure should be as follows:



- Definition 0.3.14: "there exists $x \in A$ such that $f(x) = z$" should be "there exists $x \in D$ such that $f(x) = z$."
- Section 4=0.4.4: "...the cryptographer changes the scheme simply by removing ♠ as a possible value for p" should be "... as a possible value for k."
- Section 0.5.4: At the end of the section labeled *Mutating a set*,

```
>>> U=S.copy()
>>> U.add(5)
>>> S
{1, 3}
```

  should end with

```
>>> S
{6}
```

- Problem 0.8.5: "`row(p)`" should be "`row(p, n)`".
- Section 1.4.1: "Using the fact that $\mathbf{i}^2 = 1$" should be "Using the fact that $\mathbf{i}^2 = -1$"
- Section 1.4.5: The diagram illustrating rotation by 90 degrees is incorrect. The dots should form vertical lines to the left of the y-axis.
- Task 1.4.8 and 1.4.9: The figures accompanying these tasks are incorrect; they involve rotation by -90 degrees (i.e. 90 degrees clockwise) instead of 90 degrees (i.e. 90 degrees counterclockwise).
- Task 1.4.10: `image.file2image(`*filename*`)` returns a representation of a color image, namely a list of lists of 3-tuples. For the purpose of this task, you must transform it to a representation of a grayscale image, using `image.color2gray(·)`. Also, the pixel intensities are numbers between 0 and 255, not between 0 and 1. In this task, you should assign to `pts` the list of complex numbers $x + \boldsymbol{i}y$ such that the image intensity of pixel (x, y) is less than 120.
- Task 1.4.11: The task mentions `pts` but `S` is intended.
- Section 2.3: "We've seen two examples of what we can represent with vectors: multisets and sets." Actually, we've only seen multisets.

- Section 2.4.1: "or from $[-4, 4]$ to $[-3, -2]$" should be "or from $[-4, -4]$ to $[-3, -2]$".
- Example 2.6.8: For $\alpha = .75, \beta = .25$, $\alpha \boldsymbol{u}_2 + \beta \boldsymbol{v}_2$ should be $[6.25, 0]$, not $[6.25, -2]$.
- Section 2.8.3: "Here is an example of solving an instance of the 3×3 puzzle" should be "Here is an example of one step towards solving an instance of the 3×3 puzzle."
- Example 2.9.1: "Consider the dot-product of $[1, 1, 1, 1, 1]$ with $[10, 20, 0, 40, 100]$" should be "Consider the dot-product of $[1, 1, 1, 1, 1]$ with $[10, 20, 0, 40, -100]$."
- Section 2.9.2: "...in terms of five linear equations..." should be "...in terms of three linear equations...".
- Example 2.9.5:

$$cost = \text{Vec}(D, \{\text{hops} : \$2.50/ounce, \text{malt} : \$1.50/pound, \text{water} : \$0.006\}, \text{yeast} : \$0.45/gram)$$

should be

$$cost = \text{Vec}(D, \{\text{hops} : \$2.50/ounce, \text{malt} : \$1.50/pound, \text{water} : \$0.006, \text{yeast} : \$0.45/gram\})$$

- Definition 2.9.6: "A *linear equation* is an equation of the form $\boldsymbol{a} \cdot \ = \beta$, where ... is a vector variable." should be "A *linear equation* is an equation of the form $\boldsymbol{a} \cdot \boldsymbol{x} = \beta$, where ... $\boldsymbol{x}$ is a vector variable."
- Example 2.9.7: The total energy is not 625J but is 0.0845J, as the Python shows.
- Quiz 2.9.9: The total energy consumed in the last row of the table should be 1 J, not 1 W.
- Definition 2.9.10: "In general, a *system of linear equations* (often abbreviated *linear system*) is a collection of equations:

$$\begin{aligned} \boldsymbol{a}_1 \cdot \ &= \beta_1 \\ \boldsymbol{a}_2 \cdot \ &= \beta_2 \\ &\vdots \\ \boldsymbol{a}_m \cdot \ &= \beta_m \end{aligned}$$

where is a vector variable. A *solution* is a vector ˆ that satisfies all the equations."

should be

"In general, a *system of linear equations* (often abbreviated *linear system*) is a collection of equations:

$$\begin{aligned} \boldsymbol{a}_1 \cdot \boldsymbol{x} &= \beta_1 \\ \boldsymbol{a}_2 \cdot \boldsymbol{x} &= \beta_2 \\ &\vdots \\ \boldsymbol{a}_m \cdot \boldsymbol{x} &= \beta_m \end{aligned}$$

where $\boldsymbol{x}$ is a vector variable. A *solution* is a vector $\hat{\boldsymbol{x}}$ that satisfies all the equations."

- Quiz 2.9.13: The solution should be "The dot-products are $[2, 2, 0, 0]$."
- Quiz 2.9.14: The solution should be $[14, 20, 26, 32]$.
- Quiz 2.9.15: In the solution, the range should be `range(len(haystack)-s+1)`, not `range(len(haystack)-s)`.
- Example 2.9.17:
  - "The password is ˆ = 10111" should be "The password is $\hat{\boldsymbol{x}} = 10111$",

  - “Harry computes the dot-product $\boldsymbol{a}_1 \cdot \hat{}$” should be “Harry computes the dot-product $\boldsymbol{a}_1 \cdot \hat{\boldsymbol{x}}$”
  - “Harry computes the dot-product $\boldsymbol{a}_2 \cdot \hat{}$” should be “Harry computes the dot-product $\boldsymbol{a}_2 \cdot \hat{\boldsymbol{x}}$”
  - “Carole lets Harry log in if $\beta_1 = \boldsymbol{a}_1 \cdot \hat{}, \beta_2 = \boldsymbol{a}_2 \cdot \hat{}, \ldots, \beta_k = \boldsymbol{a}_k \cdot \hat{}$.” should be “Carole lets Harry log in if $\beta_1 = \boldsymbol{a}_1 \cdot \hat{\boldsymbol{x}}, \beta_2 = \boldsymbol{a}_2 \cdot \hat{\boldsymbol{x}}, \ldots, \beta_k = \boldsymbol{a}_k \cdot \hat{\boldsymbol{x}}$.”

- Example 2.9.28: “Eve can use the distributive property to compute the dot-product of this sum with the password even though she does not know the password:

$$\begin{array}{rcccc} (01011 + 11110)\cdot & = & 01011\cdot & + & 11110\cdot \\ & = & 0 & + & 1 \\ & = & & 1 & \end{array}$$

”

should be

“Eve can use the distributive property to compute the dot-product of this sum with the password $\boldsymbol{x}$ even though she does not know the password:

$$\begin{array}{rcccc} (01011 + 11110)\cdot \boldsymbol{x} & = & 01011 \cdot \boldsymbol{x} & + & 11110 \cdot \boldsymbol{x} \\ & = & 0 & + & 1 \\ & = & & 1 & \end{array}$$

”

- Task 2.12.8: “Did you get the same result as in Task ???” should be “Did you get the same result as in Task 2.12.7?”

- Quiz 3.1.7: the solution

```
def lin_comb(vlist,clist):
  return sum([coeff*v for (c,v) in zip(clist, vlist)])
```

should be

```
def lin_comb(vlist,clist):
  return sum([coeff*v for (coeff,v) in zip(clist, vlist)])
```

- Section 3.2.4: The representation of the old generator $[0, 0, 1]$ in terms of the new generators $[1, 0, 0]$, $[1, 1, 0]$, and $[1, 1, 1]$ should be

$$[0, 0, 1] = 0\,[1, 0, 0] - 1\,[1, 1, 0] + 1\,[1, 1, 1]$$

- In Example 3.2.7, “The secret password is a vector $\hat{}$ over $GF(2)$.... the human must respond with the dot-product $\boldsymbol{a} \cdot \hat{}$.” should be “The secret password is a vector $\hat{\boldsymbol{x}}$ over $GF(2)$... the human must respond with the dot-product $\boldsymbol{a} \cdot \hat{\boldsymbol{x}}$.”

- Example 3.3.10: “This line can be represented as Span $\{[1, -2, -2]\}$” should be “This line can be represented as Span $\{[-1, -2, 2]\}$”

- In Example 3.5.1, “There is one plane through the points $\boldsymbol{u}_1 = [1, 0, 4.4]$, $\boldsymbol{u}_2 = [0, 1, 4]$, and $\boldsymbol{u}_2 = [0, 0, 3]$” should be “There is one plane through the points $\boldsymbol{u}_1 = [1, 0, 4.4]$, $\boldsymbol{u}_2 = [0, 1, 4]$, and $\boldsymbol{u}_3 = [0, 0, 3]$”.

- Section 4.1.4: The pretty-printed form of `M` should be

```
>>> print(M)
        #  @  ?
     ---------
 a  |   2  1  3
 b  |  20 10 30
```

for some order of the columns.

- Quiz 4.1.9: The given implementation of `mat2rowdict` will not work until you have implemented the `getitem` procedure in `mat.py`.

- Quiz 4.3.1: The pretty-printed form of `mat2vec(M)` should be

```
>>> print(mat2vec(M))
 ('a', '#') ('a', '?') ('a', '@') ('b', '#') ('b', '?') ('b', '@')
-------------------------------------------------------------------
          2           3           1          20          30          10
```

for some order of the columns.

- Quiz 4.4.2: The pretty-printed form of `transpose(M)` should be

```
>>> print(transpose(M))
        a  b
     ------
 #  |    2 20
 @  |    1 10
 ?  |    3 30
```

for some order of the rows. Also, in the solution, the upper-case `F` should be replaced with a lower-case `f`.

- Example 4.6.6: The matrix-vector product should be $[1, -3, -1, 4, -1, -1, 2, 0, -1, 0]$.

- Definition 4.6.9: "An $n \times n$ *upper-triangular* matrix A is a matrix with the property that $A_{ij} = 0$ for $j > i$" should be "for $i > j$."

- Section 4.7.2: "Applying Lemma 4.7.4 with $\boldsymbol{v} = \boldsymbol{u}_1$ and $\boldsymbol{z} = \boldsymbol{u}_1 - \boldsymbol{u}_2$" should be "Applying Lemma 4.7.4 with $\boldsymbol{v} = \boldsymbol{u}_2$ and $\boldsymbol{z} = \boldsymbol{u}_1 - \boldsymbol{u}_2$"

- Section 4.7.4: "because it is the same as $H * \boldsymbol{c}$, which she can compute" should be "because it is the same as $H * \tilde{\boldsymbol{c}}$, which she can compute"

- Section 4.9.3: The URL `http://xkcd.com/824` should be `ttps://xkcd.com/184/`.

- Section 4.11.2: "and here is the same diagram with the walk 3 c 2 e 4 2 shown" should be "and here is the same diagram with the walk 3 c 2 e 4 e 2 shown"

- Example 4.11.9: $g \circ f([x_1, x_2])$ should be $[x_1 + x_2, x_1 + 2x_2]$.

- Example 4.11.15: The last matrix (in the third row) should be $\begin{bmatrix} 7 & 19 \\ 4 & 8 \end{bmatrix}$. a superscript "T" indicating transpose:

$$= \begin{bmatrix} 7 & 4 \\ 19 & 8 \end{bmatrix}^T$$

- Example 4.13.15: $xvec_1$ should be x_1 and $xvec_2$ should be x_2.

- The description of Task 4.14.2 comes before the heading "Task 4.14.2".

- Section 4.15 (Geometry Lab): *position* is used synonymously with *location*.

- Section 4.14.6: "Hint: this uses the special property of the order of H's rows" should be "Hint: this uses the special property of the order of H's columns."

- Problem 4.17.10 is the same as Problem 4.17.5.
- Problem 4.17.18: "For this procedure, the only operation you are allowed to do on `A` is vector-matrix multiplication, using the `*` operator: `v*A`." should be "For this procedure, the only operation you are allowed to do on `B` is vector-matrix multiplication, using the `*` operator: `v*B`."
- Problem 4.17.21: $xvec_2$ should be x_2.
- Section 5.3.1: The Grow algorithm should be:

```
def GROW(𝒱)
  B = ∅
  repeat while possible:
      find a vector v in 𝒱 that is not in Span B, and put it in B.
```

- Example 5.3.2: "Finally, note that Span $B = \mathbb{R}^2$ and that neither $\boldsymbol{v}_1$ nor $\boldsymbol{v}_2$ alone could generate $\mathbb{R}^2$" should be $\mathbb{R}^3$.
- Section 5.4.3: "Let D be the set of nodes, e.g. $D = \{\text{Pembroke}, \text{Athletic}, \text{Main}, \text{Keeney}, \text{Wriston}\}$" should be "$D = \{\text{Pembroke}, \text{Athletic}, \text{Bio-Med}, \text{Main}, \text{Keeney}, \text{Wriston}, \text{Gregorian}\}$"
- Section 5.9.1: "The first vector $\boldsymbol{a}_1$ goes horizontally from the top-left corner of the whiteboard element to the top-right corner" should be "The first vector $\boldsymbol{a}_1$ goes horizontally from the top-left corner of the top-left sensor element to the top-right corner" and "The second vector $\boldsymbol{a}_2$ goes vertically from the top-left corner of whiteboard to the bottom-left corner" should be "The second vector $\boldsymbol{a}_2$ goes vertically from the top-left corner of the top-left sensor element to the bottom-left corner."

```
L = [[0,0,0],[1,0,0],[0,1,0],[1,1,0],[0,0,1],[1,0,1],[0,1,1],[1,1,1]]]
```

  should be

```
L = [[0,0,0],[1,0,0],[0,1,0],[1,1,0],[0,0,1],[1,0,1],[0,1,1],[1,1,1]]
```

- Section 5.9.1, diagram: The point in the bottom-left-back of the cube should be labeled (0,1,1) but is labeled (0,1,0).
- Section 5.9.5: In "For the third basis vector $\boldsymbol{a}_2$..." and "Remember that $\boldsymbol{a}_2$ points from the camera center to the top-left corner of the sensor array, so $\boldsymbol{a}_2 = (-.5, -.5, 1)$", a_2 should be a_3, and $a_3 = [0, 0, 1]$. The third vector in `cb` has an extra 0.
- "The third vector $\boldsymbol{c}_3$ goes from the origin (the camera center) to the top-right corner of whiteboard." should be "The third vector $\boldsymbol{c}_3$ goes from the origin (the camera center) to the top-left corner of the whiteboard."
- Section 5.12.1:
- Section 5.12.6: The vector $\begin{bmatrix} x_1 \\ xvec_2 \\ 1 \end{bmatrix}$ should be $\begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$
- Section 5.12.6: After Task 5.12.2, "Let $[y_1, y_2, y_3] = H\boldsymbol{x}$" should be "Let $[y_1, y_2, y_3] = \hat{H}\boldsymbol{x}$".
- Problem 5.14.18: "Write and test a procedure `superset_basis(S, L)`" should be "Write and test a procedure `superset_basis(T, L)`".
- Lemma 6.2.13 (Superset-Basis Lemma) states

> For any vector space $\mathcal{V}$ and any linearly independent set A of vectors, $\mathcal{V}$ has a basis that contains all of A.

but should state

> For any vector space $\mathcal{V}$ and any linearly independent set A of vectors belonging to $\mathcal{V}$, $\mathcal{V}$ has a basis that contains all of A.

- Example 6.3.3: $\mathcal{V}$ is defined to be the null space of $\begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$ but should be defined to be the null space of $\begin{bmatrix} 0 & 1 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$.

- Problem 6.7.3: The output condition says

  for $i = 1, 2, \ldots, k$,

  $$\text{Span } S = \text{Span } S \cup \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_i\} - \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_k\}$$

  but should say

  for $i = 1, 2, \ldots, k$,

  $$\text{Span } S = \text{Span } S \cup \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_i\} - \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_i\}$$

- Section 7.7.1: $xvec_1$ and $xvec_2$ should be x_1 and x_2

- Section 7.7.4: "Generating $mathbfu$" should be "Generating $\mathbf{u}$.

- Section 7.8.3: "We can represent the factorization of 1176 by the list $[(2, 3), (5, 2)]$, indicating that 1176 is obtained by multipying together three 2's and two 5's" should be "We can represent the factorization of 1176 by the list $[(2, 3), (3, 1), (7, 2)]$, indicating that 1176 is obtained by multiplying together three 2's, one 3 and two 7's", and "$1176 = 2^3 5^2$" should be "$1176 = 2^3 3^1 7^2$".

- Task 7.8.7: For $x = 61$, the factored entry has $2 \cdot 3 \cdot \cdot 7 \cdot 13$. This should be $2 \cdot 3 \cdot 7 \cdot 31$.

- Task 7.8.9: "$\texttt{gcd}(a, b)$" should be "$\texttt{gcd}(a - b, N)$".

- Section 9.2: In new spec for `project_orthogonal(b, vlist)`, output should be "the projection $\boldsymbol{b}^\perp$ of $\boldsymbol{b}$ orthogonal to the vectors in *vlist*"

- Example 9.4.1: The math is misformatted; there should be a line-break just before $\boldsymbol{b}_2$. That is, the math should state that $\boldsymbol{b}_1 = [-1, -3.5, 0.5]$ and that $\boldsymbol{b}_2 = \boldsymbol{b}_1 - \frac{\langle \boldsymbol{b}_0, \boldsymbol{v}_2^* \rangle}{\langle \boldsymbol{v}_2^*, \boldsymbol{v}_2^* \rangle} \boldsymbol{v}_2^* = \boldsymbol{b}_1 - \frac{-1}{2}[0, 3, 3] = [-1, -2, 2]$.

- Section 9.6.6: "These vectors span the same space as input vectors $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_k, \boldsymbol{w}_1, \ldots, \boldsymbol{w}_n^*$ ...." The * in $\boldsymbol{w}_n^*$ should not be there.

- Section 9.6.6: In the pseudocode for *find_orthogonal_complement*, the last line should be Return

- Proof of Lemma 10.6.2: The first line of the last sequence of equations,

  $$\omega^{r-c} = ((\omega^{r-c})^0 + (\omega^{r-c})^1 + (\omega^{r-c})^2 + \cdots + (\omega^{r-c})^{n-2} + (\omega^{r-c})^{n-1})$$

  should be

  $$\omega^{r-c} z = \omega^{r-c}((\omega^{r-c})^0 + (\omega^{r-c})^1 + (\omega^{r-c})^2 + \cdots + (\omega^{r-c})^{n-2} + (\omega^{r-c})^{n-1})$$

- Task 10.9.16: The procedure `image_round` should also ensure the numbers are between 0 and 255.

- Proof of Lemma 11.3.6: "Let $\mathcal{V}^*$ be the space dual to $\mathcal{V}$" should be "Let $\mathcal{V}^*$ be the annihilator of $\mathcal{V}$", and "the dual of the dual" should be "the annihilator of the annihilator".

- Section 11.3.3: "...we provide a module `svd` with a procedure `factor(A)` that, given a Mat `A`, returns a triple `(U, Sigma, V)` such that `A = U * Sigma * V.transpose`" should end "such that `A = U * Sigma * V.transpose()`"

- Proof of Lemma 11.3.11: "which equals $\left(\|\boldsymbol{a}_1\|^2 + \cdots + \|\boldsymbol{a}_m\|^2\right) + \left(\|\boldsymbol{a}_1^{\|\mathcal{V}}\|^2 + \cdots + \|\boldsymbol{a}_m^{\|\mathcal{V}}\|^2\right)$" should be "which equals $\left(\|\boldsymbol{a}_1\|^2 + \cdots + \|\boldsymbol{a}_m\|^2\right) - \left(\|\boldsymbol{a}_1^{\|\mathcal{V}}\|^2 + \cdots + \|\boldsymbol{a}_m^{\|\mathcal{V}}\|^2\right)$"

- Section 11.3.5, Proof of Theorem 11.3.12: There is a corrected proof at `http://codingthematrix.com/proof-that-first-k-right-singular-vectors-span-closest-space0.pdf`

- Section 11.3.10: There is a corrected proof at `http://codingthematrix.com/proof-that-U-is-column-orthogonal0.pdf`.

- Task 11.6.6, "To help you debug, applying the procedure to with" should be "To help you debug, applying the procedure with"

- Section 11.4.1: The procedure `SVD_solve`(A) should take the vector $\boldsymbol{b}$ as a second argument: `SVD_solve`(A, b).

- Section 11.6 (Eigenfaces Lab): `{x,y for x in range(166) for y in range(189)}` should be `{(x,y) for x in  range(166) for y in range(189)}`.

- Section 12.1.2: The diagonal matrix Λ is used shortly before it is defined.

- Problem 12.14.8: Error in statement of Lemma 12.14. The eigenvalue of A having smallest asolute value is the *reciprocal* of the eigenvalue of A^{-1} having largest absolute value.

- Section 12.8.1: $xvec_2^{(t)}$ should be just $x_2^{(t)}$.

- Section 12.8.1: In the equation

$$\begin{bmatrix} x_1^{(t)} \\ x_2^{(t)} \end{bmatrix} = \left(S\lambda S^{-1}\right)^t \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix}$$

  λ should be Λ.

- Section 12.8.1: $xvec_2(t)$ should be $x_2^{(t)}$ and $xvec_2^{(0)}$ should be $x_2^{(0)}$.

- Section 12.8.4: "Once consecutive addresses have been requested in timesteps t and $t+1$, it is very likely that the address requested in timestep $t+1$ is also consecutive" should end "that the address requested in timestep $t+2$ is also consecutive."

- Section 12.12.1: "The theorem in Section 12.8.2...." There is no theorem in that section; the theorem (the Perron-Frobenius Theorem) is not stated in the text.

- Section 12.12.3: The eigenvector given for the test case for Task 12.12.3 is wrong; the correct eigenvector is roughly `{1: 0.5222, 2: 0.6182, 3: 0.5738, 4: 0.0705, 5: 0.0783, 6: 0.0705}`.